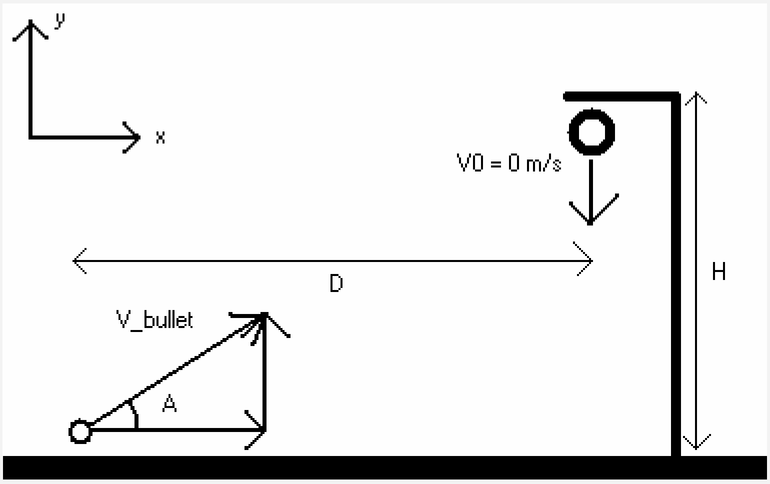
4.5 Hunter & Monkey Problem

This is a famous problem given to students when they start doing two-dimensional motion, and it usually fools them. The question goes as follows:

A monkey is hanging off a branch in the forest. A hunter aims his rifle directly at the monkey and fires. Just as he fires, the monkey lets go of the branch and starts falling to the ground. Will the bullet pass above, hit, or pass below the monkey? Assume that the monkey is close enough to the hunter so that the bullet will travel the horizontal distance between them before the monkey hits the ground.

Solution

Now when most people get this problem, they immediately assume that they don't have enough information. They want to know how far away the monkey is, or what angle the hunter is aiming his rifle at…but if you follow the normal problem-solving procedure, you will realize that you have all the information you need. Let's draw a diagram first of all:



Now let's say the problem starts just as the hunter fires and the monkey drops from the tree branch. We'll say t = 0 at this time. Now we're interested in when the bullet crosses over the vertical line the monkey is falling along. Let's say that this occurs at time t = T. We don't know very much else – we know that the monkey's initial velocity in the vertical direction as it lets go is 0 m/s. First let's find out how far the monkey falls from t = 0 to t = T. Looking at the formulas again:

$$v = v_0 + at$$

$$v_{av} = \frac{1}{2}(v_0 + v)$$

$$\Delta x = v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a\Delta x$$

We can use the third formula:

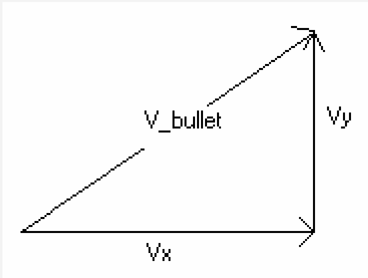
$$\Delta x = v_0 t + \frac{1}{2} a t^2$$

$$\Delta x = 0 + \frac{1}{2} \times -9.81 m/s^2 \times T^2$$

$$\Delta x = -4.905 T^2$$

Note that this is negative, since the monkey's displacement is in the negative direction. Also note that this is the monkey's displacement in the vertical direction *from where it started*, which is a distance H above where the bullet starts.

Now let's look at the movement of the hunter's bullet. First let's define some vectors for the movement of the bullet:



Now the horizontal component of velocity will stay constant, whereas the vertical component will vary. Let's find out how long it takes for the bullet to travel the horizontal distance D:

Time taken = distance/speed
$T = D/V_x$

Now what about how far the bullet travels in the vertical direction in this time T. We'll use the same equation we've used before:

$$\Delta x = v_0 t + \frac{1}{2} a t^2$$

$$\Delta x = V_y \times T - \frac{1}{2} \times 9.81 m/s^2 \times T^2$$

$$\Delta x = V_y T - 4.905 T^2$$

Remember this is an expression for how far the bullet travels in the vertical direction from *where the bullet started*. So now we have two expressions: one for how far in the vertical direction the bullet travels, and one for the monkey. We can draw a diagram for this at time t = T. Note that since we have drawn the vector representing the monkeys displacement in the negative direction, we don't need the negative sign:

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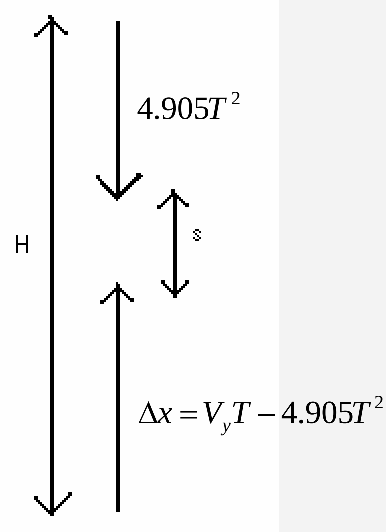

The distance between, s, can be determined as:

$$s = H - 4.905T^2 - \left(V_y T - 4.905T^2\right)$$
$$s = H - 4.905T^2 - V_y T + 4.905T^2$$
$$s = H - V_y T$$

Now we don't know any of these variables. What we can do is substitute in for some of these variables. For instance instead of T, we can write D/V_x.

$$s = H - V_y\left(\frac{D}{V_x}\right)$$

We can write H in terms of the angle the hunter fires at and the horizontal distance to the monkey, D:

$$\tan A = \frac{H}{D}$$
$$H = D \times \tan A$$
$$s = D \times \tan A - V_y\left(\frac{D}{V_x}\right)$$

But we also know that:

$$\tan A = \frac{V_y}{V_x}$$
$$s = D \times \frac{V_y}{V_x} - V_y\left(\frac{D}{V_x}\right)$$
$$s = 0$$

So the distance between the monkey and the bullet when is zero – the bullet hits the monkey. Notice that this did not depend on the angle or bullet velocity or distance to the monkey being any specific value – the bullet will hit the monkey regardless. Take an extreme example for instance. If the angle A was 90 degrees the monkey would be directly overhead the hunter. Common sense tells you that the bullet would have to hit the monkey in this case.

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Hint

A very good way to test whether your final answers are correct is to substitute very large or very small numbers into your expression and see whether they make sense. For instance, the expression for the area of a circle - $A = \pi r^2$. For a very large value of r, the area is very large – this makes sense. Or say I have an expression for the maximum speed of my car based on how much weight it is carrying - $V_{max}\ (\text{m/s}) = \frac{100}{\text{Weight in kg}}$. If I substitute an absolutely huge weight – say 100 000 kg into the expression, I get a maximum speed of 0.01 m/s. This makes sense as I would not expect the car to move very fast with such a huge weight!

4.6 Free Fall With Air Resistance

Say you jump out of a plane high above the earth. You start accelerating as you fall, and continue to do so for quite a few seconds. However, as you approach about 220 km/hr, you start to stop accelerating, and eventually you cease to accelerate at all – you are now falling at a constant velocity. You may have heard of this being 'terminal velocity'. You stop accelerating because of air resistance. The faster you fall, the larger the force slowing you down from air resistance. As you get faster and faster, eventually this air resistance force exactly counterbalances the downwards force from gravity – you stop accelerating and maintain a constant velocity. The graph of any object free-falling in an atmosphere or even in a fluid is as follows:

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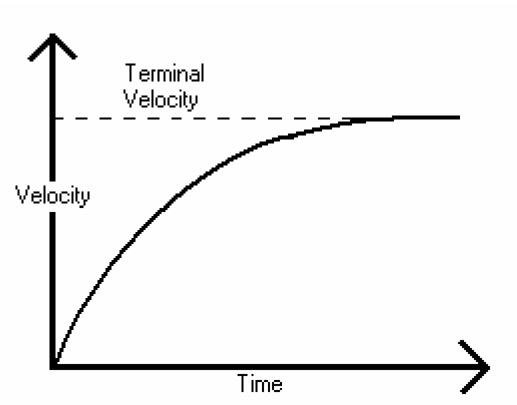

This graph has been drawn assuming positive velocities are in the *downwards* direction.

Note how the velocity of the object increases most quickly at the start. As the velocity approaches terminal velocity, the air resistance force is almost as big as the gravity force, so the acceleration is small. Eventually, the two forces balance out and a constant 'terminal velocity' is achieved, which is where the curve straightens into a horizontal line.

4.7 Spacial Dimensions

There are three commonly talked about dimensions in physics. A straight, very thin line is one-dimensional. You could describe movement along a straight line as one-dimensional movement.