The Quadratic Equation

This is what someone would call a typical quadratic equation.

$$x^2 + 3x + 2 = 0$$

Left hand side   Right hand side

This equation simply says that "x squared plus three lots of x plus two must equal zero." Now obviously, some values of x will make the equation *true* and some won't. For instance, if we make x equal 2, then:

$$2^2 + (3 \times 2) + 2 = 12 \neq 0$$

The equals sign with the line through it means "not equal to."

When someone *solves* a quadratic equation, what they are doing is finding the values of x which make the quadratic (equation) *true*. *True* means the left hand side equals the right hand side. For the same equation, what about if x = -1? Then:

$$(-1)^2 + 3 \times (-1) + 2 = 0$$

Left hand side

Remember that (-1) squared is *positive* 1. This time the left hand side does equal 0 (the right hand side), so what we have done is found a *solution* to this quadratic equation. The solution is x = -1.

One of the important things to know about quadratic equations is that they often have *two* solutions. For the same equation what if x = -2? Then:

$$(-2)^2 + 3 \times (-2) + 2 = 0$$

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The left hand side is again equal to zero! So this quadratic equation has *two* solutions, x = -1 and x = -2. Don't worry about how we got those solutions, we'll get to that soon.

Now what happens if you get an equation like this:

$$6x^2 + 3 = -3x + 2$$

This doesn't look like a quadratic equation does it? But it is, if we *rearrange* it. The basic thing to keep in mind is that *whatever you do to one side must be done to the other side as well.* This is what we do for this equation:

Subtract 2 from both sides. That gives us:

$$6x^2 + 1 = -3x$$

Now add 3x to both sides:

$$6x^2 + 3x + 1 = 0$$

This is now in *standard* quadratic equation form. Now it does look like the first equation, although with different numbers.

Anything that looks like these is likely to be a quadratic equation. The *general* form of a quadratic equation is:

$$ax^2 + bx + c = 0$$

The letters a, b, and c are just simply numbers. They can be fractions or decimals, negative or positive numbers. Any equation that fits this general form **or** can be rearranged to fit this general form is a quadratic equation. Note that a, b and c can also be zero. For instance, the following is a quadratic equation:

$$x^2 = 4$$

It doesn't look the same simply because this equation has 'b' being 0. It is really:

$$\left(1 \times x^2\right) + (0 \times x) - 4 = 0$$

If a = 0 the equation is simplified from a quadratic equation into a linear equation – one that has no powers of squared or higher.

These types of quadratics are the easiest to solve – the answer in this case is just ±2. Remember the plus or minus – the answer is both 2 **and** –2. Don't confuse this with:

$$\sqrt{4} = 2$$

The square root of 4 is just 2, <u>not</u> –2.

You only get plus or minus answers when you have the following situation:

$$(something)^2 = a\ number$$
$$something = \pm\sqrt{a\ number}$$

If only c = 0 then you could get an equation like:

$$x^2 + 3x = 0$$

This case has an obvious solution => x = 0 makes it true. To find the other solution, divide both sides by x (remember 0 divided by anything is just 0):

$$x + 3 = 0$$
$$Then:$$
$$x = -3$$

So this gives the two solutions x = -3, 0.

Another situation in which a quadratic equation occurs is the following:

$$x^4 + 2x^2 + 3 = 0$$

Now a quadratic equation must have no higher powers than a squared term. This one has a term to the fourth power. But what if we make up a letter $X = x^2$? Then we can rewrite this equation as

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$$X^2 + 2X + 3 = 0$$

Now this is in the standard quadratic equation form. In general, it is good to have an equation in this form because they can almost always be solved quickly. So you would solve this equation for X, then solve for x in $x^2 = X$. We'll deal next with ways to solve quadratic equations.

The Quadratic Formula

This is a clever formula that you can use on a quadratic equation that will almost always solve it. It seems a bit complicated at first, but is very easy to use once you get used to it. It runs as follows:

If you have a quadratic equation in the *general* form:

$$ax^2 + bx + c = 0$$

Then to find the solutions, use the values of a, b and c in the following formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

To help you understand it, let's use it on the first quadratic equation we saw:

$$x^2 + 3x + 2 = 0$$

For this equation, a = 1, b = 3 and c = 2. So:

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